CALCULUS COMPREHENSIVE EXAM

Summer 2003, Prepared by Dr. Robert Gardner

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NAME ______________________________ STUDENT NUMBER ________________________

Be clear and **give all details**. Use all symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. **No calculators!** You may omit one problem from numbers 1 through 6 (which contain Calculus 1 material) and one problem from numbers 7 through 12 (which contain Calculus 2 material). Indicate which two problems you are omitting: ______ and ______. There is a three hour time limit.

1. Do each of the following **(4)**:

   a,b. Suppose the interval $[a, b]$ is a subset of the domain of the function f. What is the definition of "f is continuous at point c where $c \in (a, b)$"? What is the definition of "f is continuous on the interval $[a, b]$"?

   c. Use the definitions from **a** and **b** to show that $f(x) = \sqrt{1 - x^2}$ is continuous on $[-1, 1]$.

2. Do each of the following **(8,10,31,34)**:

   a. State the Chain Rule (with all hypotheses).

   b. What does it mean for $f(x)$ to be implicit to the equation $F(x, y) = 0$?

   c. Differentiate (you need not simplify your answer): $f(x) = \ln \sqrt{\dfrac{\cot(e^x)}{\arctan(x) + x^2}}$.

3. Do each of the following **(14)**:

   a. Clearly state the relationship between the increasing-ness/decreasing-ness of a function and its first derivative.

   b. Find the intervals on which $f(x) = x^{1/3}(x - 4)$ is increasing/decreasing and graph f.

4. Sand falls from a conveyor belt at the rate of 10 m^3/min onto the top of a conical pile. The height of the pile is always three-eighths of the base diameter. How fast is the height changing when the pile is 4 m high? HINT: The volume V of a (right circular) cone of base radius r and height h is $V = \dfrac{1}{3}\pi r^2 h$. **(23, 24)**

5. State both parts of the Fundamental Theorem of Calculus. Evaluate $\displaystyle\int_1^2 \frac{(\ln x)^7}{x}\,dx$ and indicate with a star $(*)$ where you have used the Fundamental Theorem of Calculus in your computation. **(23, 24)**

6. Do each of the following:

   a. State the definition of *partition*, *norm* of a partition, *Riemann sum*, and *definite integral* for $\int_a^b f(x)\,dx$.

   b. Explain the difference between a definite integral and an indefinite integral (if any). (**20, 21, 23**)

7. Find the length of $y = x^2$ for $x \in [0, 1]$. (**27, 34**)

8. The region bounded by the positive x-axis, the positive y-axis, and $y = e^{-x}$ is revolved about the y -axis. What's the volume? (**26, 31, 38**)

9. State the definition of $g(x) = \tan^{-1} x$. Use this definition and the differentiation properties of $f(x) = \tan x$ to show that $g'(x) = \frac{d}{dx}[\tan^{-1} x] = \frac{1}{1+x^2}$. **(28, 34, 35)**

10. State the Integral Test (which concerns the convergence of a positive term series). Show that for $0 < p < 1$, the p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ diverges. **(38, 43)**

11. Do each of the following

    a. For a given x value, the power series $\sum_{n=0}^{\infty} c_n(x-a)^n$ may converge conditionally, converge absolutely, or diverge. Describe the possible behavior of this series (i.e. on what types of sets might the series converge conditionally, converge absolutely, or diverge)?

    b. What is the radius of convergence of $\sum_{n=0}^{\infty} \frac{3^n x^n}{n!}$? (**46**)

12. Compute a Taylor series for e^{-x^2} and $\int_0^x e^{-t^2}\,dt$. **(47)**