# CALCULUS COMPREHENSIVE EXAM

Spring 2004, Prepared by Dr. Robert Gardner

April 16, 2004

NAME ______________________________ STUDENT NUMBER ________________________

Be clear and **give all details**. Use symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. **No calculators!** You may omit one problem from numbers 1 through 5 (which contain Calculus 1 material) and one problem from numbers 6 through 10 (which contain Calculus 2 material). Indicate which two problems you are omitting: _____ and _____. There is a three hour time limit.

**1.** Do each of the following:

**(a)** State the definition of the limit of a function (that is, what does $\lim_{x\to a} f(x) = L$ mean?).

**(b)** Use the definition of limit to prove that $\lim_{x\to a} mx + b = ma + b$. **(1)**

**2.** Do each of the following **(8, 10, 31, 34)**:

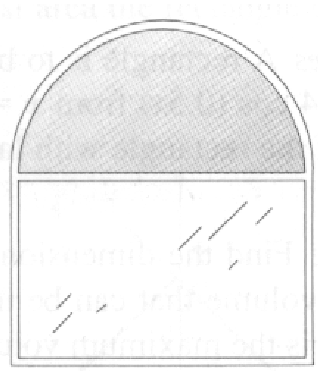
**(a)** State the Chain Rule (with all hypotheses).

**(b)** What does it mean for $f(x)$ to be implicit to the equation $F(x, y) = 0$?

**(c)** Differentiate (you need not simplify your answer): $f(x) = \ln \sqrt{\dfrac{\cot(e^x)}{\arctan(x) + x^2}}$.

**3.** State the Extreme Value Theorem for Continuous Functions. Consider $f(x) = x^{2/3}$ on the interval $[-2, 3]$. Find the absolute extrema of $f$ on this interval. **(8, 12, 16)**

**4.** A window is in the form of a rectangle surmounted by a semicircle. The rectangle is of clear glass, whereas the semicircle is of tinted glass that transmits only half as much light per unit area as clear glass does. The total perimeter is fixed. Find the proportions of the window that will admit the most light. **(18)**



**5.** Do each of the following:

**(a)** State the Fundamental Theorem of Calculus (both parts). **(23)**

**(b)** Use the Fundamental Theorem of Calculus to evaluate $\int_0^1 \frac{1+x}{1+x^2}\,dx$ and indicate with a star ($*$) where you are applying the Fundamental Theorem. **(24, 35)**

**6.** Find the length of $y = x^2$ for $x \in [0, 1]$. **(27, 34)**

**7.** Evaluate **(37, 38, 39)**:

**(a)** $\lim_{x \to 0^+} x^x$.

**(b** $\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx$ (use all notation correctly and don't write things that don't make sense).

**(c)** $\int_{-\infty}^{\infty} \frac{1}{x^2} dx$.

**8.** Do each of the following **(41)**:

**(a)** Let $\{a_n\} = \{a_1, a_2, a_3, \ldots\}$ be a sequence of real numbers. Define "$\lim_{n \to \infty} a_n = L$."

**(b)** Let $\sum_{n=1}^{\infty} a_n$ be a series. Define *partial sum* of the series and define "$\left(\sum_{n=1}^{\infty} a_n\right) = L$."

**9.** Do each of the following:

**(a)** For a given $x$ value, the power series $\sum_{n=0}^{\infty} c_n(x-a)^n$ may converge conditionally, converge absolutely, or diverge. Describe the possible behavior of this series (i.e. on what types of sets might the series converge conditionally, converge absolutely, or diverge)?

**(b)** What is the radius of convergence of $\sum_{n=0}^{\infty} \frac{3^n x^n}{n!}$? **(46)**

**10.** Find the Maclaurin Series for $f(x) = e^x$ (show your work). Where does the series converge absolutely? Where does it converge conditionally? Where does it diverge? Use the series to verify that $\int e^x\,dx = e^x + C$. **(31, 45, 46, 47)**