CALCULUS COMPREHENSIVE EXAM

Spring 2002, Prepared by Dr. Robert Gardner

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NAME ______________________________ STUDENT NUMBER ______________________

Be clear and **give all details**. Use all symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. You may omit one problem from numbers 1 through 6 which contains Calculus 1 material) and one problem from numbers 7 through 12 (which contains Calculus 2 material). Indicate which two problems you are omitting: ______ and ______.

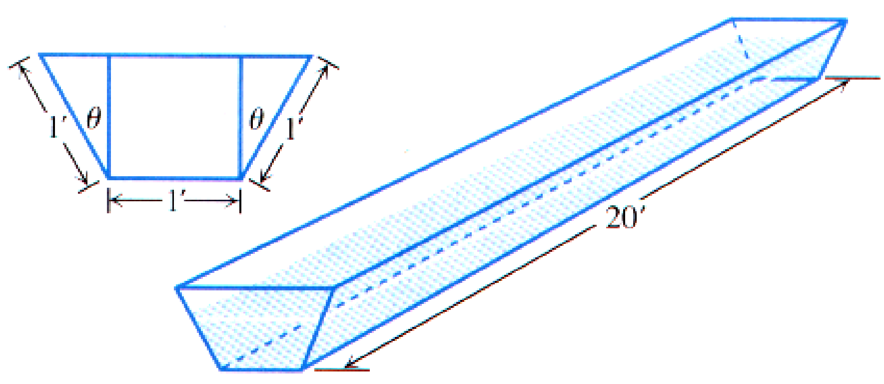
1. State the Sandwich Theorem (sometimes called the "Squeeze Theorem") for the limit of a function. Use the facts that $\sin\theta < \theta < \tan\theta$ for $\theta \in (0, \pi/2)$ to show that $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$ (WARNING: This is a two sided limit and the inequality is only known to hold for $\theta \in (0, \pi/2)$.) **(3, 33)**

2. Prove that if f has a derivative at $x = c$, then f is continuous at $x = c$. **(4, 7)**

3. What does it mean for a function $y = g(x)$ to be *implicit* to an equation $F(x, y) = 0$? For the equation $x^2 + y^2 = 1$, find y' and find 3 functions implicit to this equation. **(10)**

4. State the Extreme Value Theorem for Continuous Functions. Consider $f(x) = x^{2/3}$ on the interval $[-2, 3]$. Find the absolute extrema of f on this interval. **(8, 12, 16)**

5. The trough in the figure is to be made to the dimensions shown. Only the angle θ is to be varied. What value of θ will maximize the trough's volume? **(16, 18, 34)**



6. State both parts of the Fundamental Theorem of Calculus. Evaluate $\int_1^2 \frac{(\ln x)^7}{x}\,dx$ and indicate with a star (*) where you have used the Fundamental Theorem of Calculus in your computation. **(23, 24, 31)**

7. The region bounded by the positive x-axis, the positive y-axis, and $y = e^{-x}$ is revolved about the y-axis. What's the volume? **(26, 31, 38)**

8. State the definition of $\ln x$. Use the definition to prove that $\ln \frac{a}{b} = \ln a - \ln b$ $(b \neq 0)$. **(23, 29)**

9. State L'Hôpital's Rule for an ∞/∞ indeterminate form. Use L'Hôpital's Rule to show $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x = e$. **(31, 37)**

10. Evaluate $\int_{-1}^{1} x^{-4/3}\, dx$. WARNING: Be careful — $f(x) = x^{-4/3}$ is not continuous on $[-1, 1]$. **(23, 39)**

11. State the Integral Test (which concerns the convergence of a positive term series). Show that for $p > 1$, the p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges. **(38, 43)**

12. Consider the power series $\sum_{n=1}^{\infty} c_n (x - a)^n$. State the definition of radius of convergence. Where does the series converge absolutely? diverge? converge conditionally? **(45, 46)**