CALCULUS COMPREHENSIVE EXAM

Spring 2013, Prepared by Dr. Robert Gardner

February 1, 2013

NAME ________________________ Start time: _________ End time: _________

Be clear and **give all details**. Use symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. You may omit one problem from numbers 1 through 5 (which contain Calculus 1 material) and one problem from numbers 6 though 10 (which contain Calculus 2 material). Indicate which two problems you are omitting: ______ and ______. There is a three hour time limit. No calculators (this is a *math* test)!

1. De each of the following:

(a) State the definition of the limit of a function (i.e., what does $\lim_{x\to a} f(x) = L$ mean?). **(1)**

(b) Prove that if $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$ then $\lim_{x\to a}(f(x) + g(x)) = L + M$. **(1, 2)**

2. Prove that if $f(x)$ has a derivative at $x = c$, then f is continuous at $x = c$. Is the converse also true? **(4, 7)**

3. Do each of the following:

(a) What does it mean for $f(x)$ to be implicit to the equation $F(x, y) = 0$? **(10)**

(b) Find y' if $\sin(xy) = \ln(x \cot y)$. **(8, 31)**

4. Do each of the following:

(a) State the Fundamental Theorem of Calculus (both parts). **(23)**

(b) Use the Fundamental Theorem of Calculus to evaluate $\int_0^e \ln x\, dx$ (HINT: Use parts) and indicate with a star (*) where you are applying the Fundamental Theorem. **(23, 24, 31)**

5. Do each of the following:

(a) State the definition of *partition*, *norm* of a partition, *Riemann sum*, and *definite integral* for $\int_a^b f(x)\, dx$. **(21)**

(b) Explain the difference between a definite integral, an indefinite integral, and an antiderivative (if any). **(20, 23)**

6. Do each of the following:

(a) Use the definition of $y = \tan^{-1} x$ (in terms of the tangent function) and implicit differentiation to find $y' = \dfrac{d}{dx}[\tan^{-1} x]$. **(10, 28, 35)**

(b) Find the length of the curve given by the equation $y = \int_0^x \sqrt{\sec^4 t - 1}\, dt$ for $-\pi/4 \leq x \leq \pi/4$. **(23, 27)**

7. Do each of the following (respect the calculus!):

(a) Evaluate $\displaystyle\int_{-\infty}^{\infty} \frac{2x\, dx}{(x^2+1)^2}$. **(24, 38)**

(b) Evaluate $\displaystyle\int_{-1}^{1} \frac{1}{x^2}\, dx$. **(38, 39)**

8. Do each of the following:

(a) For a given x value, the power series $\displaystyle\sum_{n=0}^{\infty} c_n(x-a)^n$ may converge conditionally, converge absolutely, or diverge. Describe the possible behavior of this series (i.e. on what types of sets might the series converge conditionally, converge absolutely, or diverge)? **(46)**

(b) What is the radius of convergence of $\displaystyle\sum_{n=0}^{\infty} \frac{3^n x^n}{n!}$? **(45)**

9. State the Integral Test (which concerns the convergence of a positive term series). Show that for $p > 1$ the p-series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges. **(38, 43)**

10. Do each of the following:

(a) Determine whether $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1}$ converges or diverges and explain. **(43)**

(b) Use the MacLaurin Series for e^x to find a series for $\int e^{-x^2} dx$. **(30, 46)**