CALCULUS COMPREHENSIVE EXAM

Spring 2005, Prepared by Dr. Robert Gardner

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NAME ________________________ STUDENT NUMBER ________________________

Be clear and **give all details**. Use symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. **No calculators!** You may omit one problem from numbers 1 through 5 (which contain Calculus 1 material) and one problem from numbers 6 though 10 (which contain Calculus 2 material). Indicate which two problems you are omitting: ______ and ______. There is a three hour time limit.

1. Do each of the following:

(a) State the definition of the limit of a function (i.e., what does $\lim_{x\to a} f(x) = L$ mean?). **(1)**

(b) Prove that if $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$, then $\lim_{x\to a}(f(x) + g(x)) = L + M$. **(2)**

2. Do each of the following **(8, 10, 31, 34)**:

(a) State the Chain Rule (with all hypotheses).

(b) What does it mean for $f(x)$ to be implicit to the equation $F(x, y) = 0$?

(c) Differentiate (you need not simplify your answer): $f(x) = \ln \sqrt{\dfrac{\cot(e^x)}{\arctan(x) + x^2}}$.

3. Do each of the following **(14)**:

(a) Clearly state the relationship between the increasing-ness/decreasing-ness of a function and its first derivative.

(b) Find the intervals on which $f(x) = x^{1/3}(x - 4)$ is increasing/decreasing and graph f.

4. The intensity of illumination at any point is proportional to the product of the strength of the light source and the inverse of the square of the distance from the source. If two sources of relative strengths a and b are a distance c apart, at what point on the line joining them will the intensity be minimum? Assume the intensity at any point is the sum of intensities from the two sources.

5. Do each of the following:

(a) State the definition of *partition*, *norm* of a partition, *Riemann sum*, and *definite integral* for $\int_a^b f(x)\,dx$. **(21)**

(b) State the two parts of the Fundamental Theorem of Calculus.

6. **(a)** Use the definition of $y = \tan^{-1} x$ (in terms of the tangent function) and implicit differentiation to find $y' = \dfrac{d}{dx}[\tan^{-1} x]$.

(b) Evaluate $\displaystyle\int \frac{dx}{x^2 - 2x + 5}$. **(28, 34, 35)**

7. Do each of the following **(38, 39, 41)**:

(a) Evaluate $\displaystyle\int_{-\infty}^{\infty} \frac{2x\,dx}{(x^2+1)^2}$.

(b) Evaluate $\displaystyle\int_{-1}^{1} \frac{1}{x^2}\,dx$.

(c) Evaluate $\displaystyle\int \frac{8}{(4x^2+1)^2}\,dx$. **(24, 35)**

8. Do each of the following.

(a) State the definition of the limit of a sequence: $\lim_{n\to\infty} a_n = L$. **(41)**

(b) State the definition of the sum of a series: $\displaystyle\sum_{n=1}^{\infty} a_n = S$. **(41)**

(c) Evaluate $\displaystyle\sum_{n=1}^{\infty} \left(1 - \frac{1}{2^n}\right)$.

9. Do each of the following:

(a) For a given x value, the power series $\displaystyle\sum_{n=0}^{\infty} c_n(x-a)^n$ may converge conditionally, converge absolutely, or diverge. Describe the possible behavior of this series (i.e. on what types of sets might the series converge conditionally, converge absolutely, or diverge)?

(b) For what values of x does $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1}\frac{x^n}{n}$ converge? **(46)**

10. Compute a Taylor series for e^{-x^2} and $\displaystyle\int_0^x e^{-t^2}\,dt$.