CALCULUS COMPREHENSIVE EXAM

Fall 2015b, Prepared by Dr. Robert Gardner

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NAME _____ Start Time _____ End Time: _____

Be clear and **give all details**. Use all symbols correctly (such as equal signs) and write in complete sentences. The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. **No calculators and turn off your cell phones!** Use the paper provided and **only write on one side.** You may omit one problem from numbers 1 through 5 (which contain Calculus 1 material) and one problem from numbers 6 through 10 (which contain Calculus 2 material). Indicate which two problems you are omitting: _____ and _____.

1. (a) State the definition of the limit of a function (i.e., what does $\lim_{x\to a} f(x) = L$ mean?).

(b) Prove that if $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$, then $\lim_{x\to a}(f(x) - g(x)) = L - M$ **(1,2)**

2. Prove that if $f(x)$ has a derivative at $x = c$, then f is continuous at $x = c$. Is the converse also true? **(4, 7)**

3. Do each of the following **(5, 13)**:

(a) State the Intermediate Value Theorem.

(b) State the Mean Value Theorem.

(c) Prove that $f(x) = \sin(x) + 2x - 1$ has exactly one real root.

4. (a) State the Fundamental Theorem of Calculus (both parts). **(23)**

(b) Evaluate $\int_1^e x \ln(x)\, dx$ (HINT: Use parts) and indicate with a star (*) where you have used the Fundamental Theorem of Calculus in your computations. **(24, 31)**

5. (a) State the definition of *partition*, *norm* of a partition, *Riemann sum*, and *definite integral* for $\int_a^b f(x)\, dx$. **(21)**

(b) Evaluate $\int_0^1 \frac{1}{x}\, dx$. Use correct notation. **(38, 39)**

6. Do each of the following.

(a) State the definition of $\ln x$ (using integrals). **(29)**

(b) Use the definition from part **(a)** to *prove* that $\ln(ab) = \ln(a) + \ln(b)$. **(29)**

7. Do each of the following.

(a) If f is continuous on $[a, c) \cup (c, b]$ then state the definition of $\int_a^b f(x)\,dx$. That is, how do you integrate over a discontinuity? You may assume the usual definition for integrals of continuous functions has been established. **(39)**

(b) Evaluate $\displaystyle\int_0^2 \frac{1}{(x-1)^2}\,dx$. **(39)**

8. Do each of the following.

(a) State the definition of the limit of a sequence: $\lim_{n\to\infty} a_n = L$. **(41)**

(b) State the definition of the sum of a series: $\displaystyle\sum_{n=1}^{\infty} a_n = S$. **(41)**

(c) Determine whether the series $\displaystyle\sum_{n=1}^{\infty} ne^{-2n}$ is convergent or divergent. You may use any test, but you must check the hypothesis of any test you use. **(45)**

9. Do each of the following:

(a) For a given x value, the power series $\displaystyle\sum_{n=0}^{\infty} c_n(x-a)^n$ may converge conditionally, converge absolutely, or diverge. Describe the possible behavior of this series (i.e. on what types of sets might the series converge conditionally, converge absolutely, or diverge)? **(46)**

(b) For what values of x does $\displaystyle\sum_{n=1}^{\infty}(-1)^{n-1}\frac{(x-5)^n}{5^n(n+5)}$ converge? **(46)**

10. Compute a MacLaurin series for e^{-x^2} and $\displaystyle\int_0^x e^{-t^2}\,dt$. (**47**)