

CALCULUS COMPREHENSIVE EXAM

Fall 2014b, Prepared by Dr. Robert Gardner

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NAME _____ Start Time _____ End Time: _____

Be clear and **give all details**. Use all symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. **No calculators and turn off your cell phones!** Use the paper provided and **only write on one side**. You may omit one problem from numbers 1 through 5 (which contain Calculus 1 material) and one problem from numbers 6 through 10 (which contain Calculus 2 material). Indicate which two problems you are omitting: _____ and _____.

1. Do each of the following (**1, 2**)

- (a) State the definition of the limit of a function (i.e., what does $\lim_{x \rightarrow a} f(x) = L$ mean?).
- (b) Prove that $\lim_{x \rightarrow a} mx + b = ma + b$ for $m \neq 0$.

2. Do each of the following (**5**):

- (a) State the Intermediate Value Theorem.
- (b) Use the Intermediate Value Theorem to *prove* that $f(x) = x^5 - 5x - 1$ has a real root (be sure to include all necessary hypotheses).

3. Do each of the following (**8, 10, 31, 34**):

- (a) State the Chain Rule (with all hypotheses).
- (b) What does it mean for $f(x)$ to be implicit to the equation $F(x, y) = 0$?
- (c) Differentiate (you need not simplify your answer): $f(x) = \ln \frac{\cot(e^x)}{\arctan x}$.

4. Sand falls from a conveyor belt at the rate of $10 \text{ m}^3/\text{min}$ onto the top of a conical pile. The height of the pile is always three-eighths of the base diameter. How fast is the height changing when the pile is 4 m high? HINT: The volume V of a (right circular) cone of base radius r and height h is $V = \frac{1}{3}\pi r^2 h$. (**10, 19**)

5. Do each of the following (**21, 23**):

- (a) State the definition of *partition*, *norm* of a partition, *Riemann sum*, and *definite integral* for $\int_a^b f(x) dx$.
- (b) State the Fundamental Theorem of Calculus (both parts).

6. Find the length of the curve $y = x^2$ for $x \in [0, 1]$. **(24, 27, 34)**

7. Do each of the following:

(a) Use the definition of $y = \tan^{-1} x$ (in terms of the tangent function) and implicit differentiation to find $y' = \frac{d}{dx}[\tan^{-1} x]$. **(10, 28, 35)**

(b) Find the length of the curve given by the equation $y = \int_0^x \sqrt{\sec^4 t - 1} dt$ for $-\pi/4 \leq x \leq \pi/4$. **(23, 27)**

8. Do each of the following **(31, 38)**:

(a) State L'Hôpital's Rule for an ∞/∞ indeterminate form.

(b) Use L'Hôpital's Rule to evaluate $\lim_{x \rightarrow \infty} (1 + \frac{1}{x})$.

9. Do each of the following **(41, 46)**:

(a) State the definition of the sum of a series: $\sum_{n=1}^{\infty} a_n = S$.

(b) For a given x value, the power series $\sum_{n=0}^{\infty} c_n(x - a)^n$ may converge conditionally, converge absolutely, or diverge. Describe the possible behavior of this series (i.e. on what types of sets might the series converge conditionally, converge absolutely, or diverge)?

(c) For what values of x does $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{(x - 5)^n}{5^n(n + 5)}$ converge?

10. Find a MacLaurin Series for $f(x) = e^{2x} = e^x$ (show your work). Show and/or explain why the series converges absolutely for all x . Use the series to calculate $\lim_{x \rightarrow 0} \frac{e^{2x} - e^x}{x}$. **(31, 45, 46, 47)**