

CALCULUS COMPREHENSIVE EXAM

Fall 2013b, Prepared by Dr. Robert Gardner

December 6, 2013

NAME _____ STUDENT NUMBER _____

Be clear and **give all details**. Use all symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. **No calculators and turn off your cell phones!** You may omit one problem from numbers 1 through 5 (which contain Calculus 1 material) and one problem from numbers 6 through 10 (which contain Calculus 2 material). Indicate which two problems you are omitting: _____ and _____.

1. (a) State the definition of the limit of a function (i.e., what does $\lim_{x \rightarrow a} f(x) = L$ mean?).
(b) Prove that if $\lim_{x \rightarrow a} f(x) = L$ and $\lim_{x \rightarrow a} g(x) = M$, then $\lim_{x \rightarrow a} (f(x) - g(x)) = L - M$ **(1,2)**
2. (a) State L'Hôpital's rule.
(b) Determine $\lim_{x \rightarrow 0^+} (1 - 2x)^{3/x}$ **(37)**
3. Do each of the following **(5, 13)**:
 - (a) State the Intermediate Value Theorem.
 - (b) State the Mean Value Theorem.
 - (c) Prove that $f(x) = \sin(x) + 2x - 1$ has exactly one real root.
4. (a) State the Fundamental Theorem of Calculus (both parts). **(23)**
(b) Evaluate $\int_0^e \ln x \, dx$ (HINT: Use parts) and indicate with a star (*) where you have used the Fundamental Theorem of Calculus in your computations. **(23,24,31)**
5. (a) State the definition of *partition*, *norm* of a partition, *Riemann sum*, and *definite integral* for $\int_a^b f(x) \, dx$. **(21)**
(b) Explain the difference between a definite integral and an indefinite integral (if any). **(20, 23)**

6. Do each of the following:

(a) Use the definition of $y = \tan^{-1} x$ (in terms of the tangent function) and implicit differentiation to find $y' = \frac{d}{dx}[\tan^{-1} x]$. **(10, 28, 35)**

(b) Find the length of the curve given by the equation $y = \int_0^x \sqrt{\sec^4 t - 1} dt$ for $-\pi/4 \leq x \leq \pi/4$. **(23, 27)**

7. Do each of the following. Respect the calculus!

(a) Evaluate $\lim_{x \rightarrow 0^+} \left(1 + \frac{1}{x}\right)^x$. **(31, 37)**

(b) Evaluate $\int \frac{dx}{x^2 - 2x + 5}$. **(28, 34, 35)**

8. Do each of the following.

(a) State the definition of the limit of a sequence: $\lim_{n \rightarrow \infty} a_n = L$. **(41)**

(b) State the definition of the sum of a series: $\sum_{n=1}^{\infty} a_n = S$. **(41)**

(c) Use the Integral Test to show that a p -series with $p > 1$ converges. **(43)**

9. Do each of the following:

(a) For a given x value, the power series $\sum_{n=0}^{\infty} c_n(x-a)^n$ may converge conditionally, converge absolutely, or diverge. Describe the possible behavior of this series (i.e. on what types of sets might the series converge conditionally, converge absolutely, or diverge)? **(46)**

(b) For what values of x does $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{(x-5)^n}{5^n(n+5)}$ converge? **(46)**

10. Compute a MacLaurin series for $\sin(-x^3)$ and $\int_0^x \sin(-t^3) dt$. **(47)**