CALCULUS COMPREHENSIVE EXAM

Fall 2021b, Prepared by Dr. Robert Gardner

November 19, 2021

NAME _____ Start Time _____ End Time: _____

Be clear and **give all details**. Use all symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. **No calculators and turn off your cell phones!** Use the paper provided and **only write on one side.**

You may omit one problem from numbers 1 through 5 (which contain Calculus 1 material) and one problem from numbers 6 through 10 (which contain Calculus 2 and multiple integral material). Indicate which two problems you are omitting: _____ and _____.

1. **(a)** State the definition of the limit of a function (i.e., what does $\lim_{x\to a} f(x) = L$ mean?).

(b) Prove that if $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$, then $\lim_{x\to a}(f(x) - g(x)) = L - M$ **(1,2)**

2. Do each of the following **(5)**:

(a) State the Intermediate Value Theorem.

(b) Prove that $\cos x = x$ for some x.

3. Do each of the following **(20, 21, 23)**:

(a) State the definition of *partition*, *norm* of a partition, *Riemann sum*, and *definite integral* for $\int_a^b f(x)\,dx$.

(b) Explain the difference between a definite integral and an indefinite integral (if any).

4. **(a)** State the Fundamental Theorem of Calculus (both parts). **(23)**

(b) Use the Fundamental Theorem of Calculus to evaluate $\int_0^1 x \sin x\,dx$ and indicate with a star $(*)$ where you are applying the Fundamental Theorem. **(24)**

5. **(a)** Determine $\lim_{x\to 0^+} (1-2x)^{3/x}$. **(29,37)**

(b) Find $\dfrac{dy}{dx}$: $\tan^{-1}(\ln y) = e^{x^2}$. **(8, 10, 31, 35)**

6. Do each of the following **(29)**:

(a) State the definition of $\ln x$ (using integrals).

(b) Use the definition from part **(a)** to *prove* that $\ln(ab) = \ln(a) + \ln(b)$.

7. Do each of the following **(39, 41, 43)**:

(a) Evaluate $\int_{-\infty}^{\infty} \frac{1}{x^2}\,dx$.

(b) Use the Integral Test to show that the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges.

8. Do each of the following **(46)**:

(a) For a given x value, the power series $\sum_{n=0}^{\infty} c_n(x-a)^n$ may converge conditionally, converge absolutely, or diverge. Describe the possible behavior of this series (i.e., on what types of sets might the series converge conditionally, converge absolutely, or diverge)?

(b) What is the radius of convergence of $\sum_{n=0}^{\infty} \frac{3^n x^n}{n!}$?

9. Find a MacLaurin Series for $f(x) = e^{2x} - e^x$ (show your work). Explain why the series converges absolutely for all x. Use the series to calculate $\lim_{x\to 0} \frac{e^{2x} - e^x}{x}$. **(31, 45, 46, 47)**

10. Find the volume of the solid that is bounded above by the cylinder $z = x^2$ and below by the region enclosed by the parabola $y = 2 - x^2$ and the line $y = x$ in the xy-plane. **(49)**