CALCULUS COMPREHENSIVE EXAM

Fall 2019b, Prepared by Dr. Robert Gardner

November 22, 2019

NAME _____ Start Time _____ End Time: _____

Be clear and **give all details**. Use all symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. **No calculators and turn off your cell phones!** Use the paper provided and **only write on one side.**

You may omit one problem from numbers 1 through 5 (which contain Calculus 1 material) and one problem from numbers 6 through 10 (which contain Calculus 2 material). Indicate which two problems you are omitting: _____ and _____.

1. (a) State the definition of the limit of a function (i.e., what does $\lim_{x\to a} f(x) = L$ mean?).

(b) Prove that if $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$, then $\lim_{x\to a}(f(x) - g(x)) = L - M$ **(1,2)**

2. Do each of the following **(8, 10, 31, 34)**:

(a) State the Chain Rule (with all hypotheses).

(b) What does it mean for $f(x)$ to be implicit to the equation $F(x, y) = 0$?

(c) Differentiate (you need not simplify your answer): $f(x) = \ln \sqrt{\dfrac{\cot(e^x)}{\arctan(x) + x^2}}$.

3. Do each of the following **(14)**:

(a) Clearly state the relationship between the increasing-ness/decreasing-ness of a function and its first derivative.

(b) Find the intervals on which $f(x) = x^{1/3}(x - 4)$ is increasing/decreasing and graph f.

4. (a) State the Fundamental Theorem of Calculus (both parts). **(23)**

(b) Use the Fundamental Theorem of Calculus to evaluate $\int_0^e \ln x \, dx$ and indicate with a star $(*)$ where you are applying the Fundamental Theorem. Hint: Integrate by parts. **(23, 24, 31)**

5. (a) State the definition of *partition*, *norm* of a partition, *Riemann sum*, and *definite integral* for $\int_a^b f(x)\, dx$. **(21)**

(b) Explain the difference between a definite integral and an indefinite integral (if any). **(20, 23)**

6. **(a)** Use the definition of $y = \sin^{-1} x$ (in terms of the sine function) and implicit differentiation to find $y' = \dfrac{d}{dx}[\sin^{-1} x]$. **(35)**

(b) Evaluate $\displaystyle\int \frac{dx}{\sqrt{5 - 4x - x^2}}$. **(28, 34, 35)**

7. Do each of the following. Respect the calculus!

(a) Evaluate $\displaystyle\lim_{x \to 0^+} \left(1 + \frac{1}{x}\right)^x$. **(31, 37)**

(b) Evaluate $\displaystyle\int_{-1}^{1} \frac{1}{x^2}\, dx$. **(39)**

8. Do each of the following.

(a) State the definition of the limit of a sequence: $\displaystyle\lim_{n \to \infty} a_n = L$. **(41)**

(b) State the definition of the sum of a series: $\displaystyle\sum_{n=1}^{\infty} a_n = S$. **(41)**

(c) Evaluate $\displaystyle\sum_{n=1}^{\infty} \left(1 - \frac{1}{2^n}\right)$. **(45)**

9. Do each of the following:

(a) For a given x value, the power series $\displaystyle\sum_{n=0}^{\infty} c_n (x - a)^n$ may converge conditionally, converge absolutely, or diverge. Describe the possible behavior of this series (i.e. on what types of sets might the series converge conditionally, converge absolutely, or diverge)? **(46)**

(b) For what values of x does $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$ converge? **(46)**

10. Do each of the following:

(a) Use the MacLaurin Series for e^x to find a series for $\displaystyle\int e^{-x^2} dx$. **(30, 46)**

(b) Estimate $\displaystyle\int_0^1 e^{-x^2} dx$ to the nearest 0.001 and explain why you know your answer has this level of accuracy. **(44, 47)**