CALCULUS COMPREHENSIVE EXAM

Fall 2018a, Prepared by Dr. Robert Gardner

September 14, 2018

NAME ________________________________ Start Time ________ End Time: ________

Be clear and **give all details**. Use all symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. **No calculators and turn off your cell phones!** Use the paper provided and **only write on one side.** You may omit one problem from numbers 1 through 5 (which contain Calculus 1 material) and one problem from numbers 6 through 10 (which contain Calculus 2 material). Indicate which two problems you are omitting: ______ and ______.

1. Do each of the following **(1,2)**:

(a) State the definition of the limit of a function (that is, what does $\lim_{x\to a} f(x) = L$ mean?).

(b) Prove that if $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$, then $\lim_{x\to a}(f(x) - g(x)) = L - M$.

2. Do each of the following **(5)**:

(a) State the Intermediate Value Theorem.

(b) State the Mean Value Theorem.

(c) Prove that $\cos x = x$ for some x.

3. Do each of the following **(8, 10, 31, 35)**:

(a) What does it mean for $f(x)$ to be implicit to the equation $F(x, y) = 0$?

(b) State the Chain Rule (with all hypotheses).

(c) Find $\dfrac{dy}{dx}$: $\tan^{-1}(\ln y) = e^{x^2}$.

4. Do each of the following **(20, 21, 23)**:

(a) State the definition of *partition*, *norm* of a partition, *Riemann sum*, and *definite integral* for $\int_a^b f(x)\,dx$.

(b) Explain the difference between a definite integral and an indefinite integral (if any).

5. Do each of the following**(23, 24, 35)**:

(a) State the Fundamental Theorem of Calculus (both parts). **(23)**

(b) Evaluate $\int_0^1 \tan^{-1}(x)\,dx$ (HINT: Use parts) and indicate with a star (*) where you have used the Fundamental Theorem of Calculus in your computations. **(24, 31)**

6. Do each of the following **(29)**:

(a) State the definition of $\ln x$ (using integrals).

(b) Use the definition from part **(a)** to *prove* that $\ln(ab) = \ln(a) + \ln(b)$.

7. Do each of the following **(32, 37)**:

(a) If f is continuous on $[a, c) \cup (c, b]$ then state the definition of $\int_a^b f(x)\, dx$. That is, how do you integrate over a discontinuity? You may assume the usual definition for integrals of continuous functions has been established.

(b) Evaluate $\displaystyle\int_0^2 \frac{1}{(x-1)^2}\, dx$.

8. Do each of the following **(41,43, 45)**:

(a) State the definition of the limit of a sequence: $\lim_{n \to \infty} a_n = L$.

(b) State the definition of the sum of a series: $\displaystyle\sum_{n=1}^{\infty} a_n = S$.

(c) Use the integral test to show that the harmonic series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ diverges.

9. Do each of the following **(46)**:

(a) For a given x value, the power series $\displaystyle\sum_{n=0}^{\infty} c_n(x-a)^n$ may converge conditionally, converge absolutely, or diverge. Describe the possible behavior of this series (i.e. on what types of sets might the series converge conditionally, converge absolutely, or diverge)?

(b) For what values of x does $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{(x-5)^n}{5^n(n+5)}$ converge?

10. Find a MacLaurin Series for $f(x) = e^{2x} - e^x$ (show your work). Show and/or explain why the series converges absolutely for all x. Use the series to calculate $\displaystyle\lim_{x \to 0} \frac{e^{2x} - e^x}{x}$. **(31, 45, 46, 47)**