CALCULUS COMPREHENSIVE EXAM

Fall 2015a, Prepared by Dr. Robert Gardner

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NAME _____ Start Time _____ End Time: _____

Be clear and **give all details**. Use all symbols correctly (such as equal signs) and write in complete sentences. The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. **No calculators and turn off your cell phones!** Use the paper provided and **only write on one side.** You may omit one problem from numbers 1 through 5 (which contain Calculus 1 material) and one problem from numbers 6 through 10 (which contain Calculus 2 material). Indicate which two problems you are omitting: _____ and _____.

1. (a) State the definition of the limit of a function (i.e., what does $\lim_{x\to a} f(x) = L$ mean?).

(b) Prove that if $\lim_{x\to a} f(x) = L$ then for any $k \neq 0$, $\lim_{x\to a}(kf(x)) = kL$ **(1,2)**

2. Prove that if $f(x)$ has a derivative at $x = c$, then f is continuous at $x = c$. Is the converse also true? **(4, 7)**

3. Do each of the following **(8, 10, 31, 35)**:

(a) State the Chain Rule (with all hypotheses).

(b) What does it mean for $f(x)$ to be implicit to the equation $F(x, y) = 0$.

(c) Find $\dfrac{dy}{dx}$: $\tan^{-1}(\ln y) = e^{x^2}$.

4. (a) State the Fundamental Theorem of Calculus (both parts). **(23)**

(b) Evaluate $\displaystyle\int_1^2 xe^x\,dx$ and indicate with a star (*) where you have used the Fundamental Theorem of Calculus in your computations. **(24, 31)**

5. (a) State the definition of *partition*, *norm* of a partition, *Riemann sum*, and *definite integral* for $\displaystyle\int_a^b f(x)\,dx$. **(21)**

(b) Explain the difference between a definite integral and an indefinite integral (if any). **(20, 23)**

6. Do each of the following. Use all notation correctly and don't write things that don't make sense. **(38, 39, 41)**

(a) Evaluate $\displaystyle\int_{-\infty}^{\infty} \frac{2x\,dx}{(x^2+1)^2}$.

(b) Evaluate $\displaystyle\int_{-1}^{1} \frac{1}{x^2}\,dx$.

7. (a) State L'Hôpital's Rule for an ∞/∞ indeterminate form.

(b) Use L'Hôpital's Rule to show $\displaystyle\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$. **(31, 37)**

8. (a) Let $\{a_n\} = \{a_1, a_2, a_3, \ldots\}$ be a sequence of real numbers. Define "$\displaystyle\lim_{n\to\infty}(a_n) = L$." **(41)**

(b) Let $\displaystyle\sum_{n=1}^{\infty} a_n$ be a series. Define *partial sum* of the series and define "$\displaystyle\left(\sum_{n=1}^{\infty} a_n\right) = L$." **(41)**

9. Determine whether the following series converge or diverge and explain. **(43)**

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1}$.

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(2n+1)!}$.

10. Do each of the following **(46)**:

(a) For a given x value, the power series $\displaystyle\sum_{n=0}^{\infty} c_n(x-a)^n$ may converge conditionally, converge absolutely, or diverge. Describe the possible behavior of this series (i.e., on what types of sets might the series converge conditionally, converge absolutely, or diverge)?

(b) What is the radius of convergence of $\displaystyle\sum_{n=0}^{\infty} \frac{3^n x^n}{n!}$?