CALCULUS COMPREHENSIVE EXAM

Fall 2014a, Prepared by Dr. Robert Gardner

September 19, 2014

NAME ______________________________ Start Time ________ End Time: ________

Be clear and **give all details**. Use all symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. **No calculators and turn off your cell phones!** Use the paper provided and **only write on one side.** You may omit one problem from numbers 1 through 5 (which contain Calculus 1 material) and one problem from numbers 6 through 10 (which contain Calculus 2 material). Indicate which two problems you are omitting: ______ and ______.

1. Do each of the following **(1, 2)**

(a) State the definition of the limit of a function (i.e., what does $\lim_{x\to a} f(x) = L$ mean?).

(b) Prove that if $\lim_{x\to a} f(x) = L$ then $\lim_{x\to a}(kf(x)) = kL$.

2. Do each of the following **(3)**:

(a) State the Sandwich Theorem (also called the Squeeze Theorem) for the limit of a function.

(b) Use the fact that $\sin(\theta) < \theta < \tan(\theta)$ for $\theta \in (0, \pi/2)$ to show that $\lim_{\theta\to 0} \frac{\sin(\theta)}{\theta} = 1$. WARNING: This is a two sided limit and the inequality is known to hold only for $\theta \in (0, \pi/2)$.

3. Do each of the following **(5, 13)**:

(a) State the Intermediate Value Theorem.

(b) State the Mean Value Theorem.

(c) Prove that $f(x) = \sin(x) + 2x - 1$ has exactly one real root.

4. Find the volume of the largest right circular cone that can be inscribed in a sphere of radius 3. The volume of a cone is given by $V = \frac{1}{3}\pi r^2 h$ and the volume of a sphere by $V = \frac{4}{3}\pi r^3$. **(8, 12, 16, 18)**

5. Do each of the following **(23, 24)**:

(a) State the Fundamental Theorem of Calculus (both parts).

(b) Use the Fundamental Theorem of Calculus to evaluate $\int_0^1 xe^x dx$ and indicate with a star (*) where you are applying the Fundamental Theorem.

6. Find the length of the curve $y = x^2$ for $x \in [0,1]$. **(24, 27, 34)**

7. Do each of the following **(23, 29, 31)**:

(a) State the definition of $\ln(x)$ (using integrals).

(b) Use the definition from part **(a)** to prove that $\ln(ab) = \ln(a) + \ln(b)$.

8. Do each of the following: **(37, 38, 39)**

(a) Evaluate $\displaystyle\lim_{t \to 0^+} \left(1 + \frac{1}{t}\right)^t$.

(b) Evaluate $\displaystyle\int_{-1}^{1} \frac{1}{x^2}\, dx$. **(39)**

9. Do each of the following **(41, 46)**:

(a) State the definition of the limit of a sequence: $\displaystyle\lim_{n \to \infty} a_n = L$.

(b) State the definition of the sum of a series: $\displaystyle\sum_{n=1}^{\infty} a_n = S$.

(c) For a given x value, the power series $\displaystyle\sum_{n=0}^{\infty} c_n (x-a)^n$ may converge conditionally, converge absolutely, or diverge. Describe the possible behavior of this series (i.e. on what types of sets might the series converge conditionally, converge absolutely, or diverge)?

10. Compute a MacLaurin series for e^{-x^2} and $\displaystyle\int_0^x e^{-t^2}\, dt$. **(47)**