CALCULUS COMPREHENSIVE EXAM

Summer 2015, Prepared by Dr. Robert Gardner

July 24, 2015

NAME _____ Start Time _____ End Time: _____

Be clear and **give all details**. Use all symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. **No calculators and turn off your cell phones!** Use the paper provided and **only write on one side.** You may omit one problem from numbers 1 through 5 (which contain Calculus 1 material) and one problem from numbers 6 through 10 (which contain Calculus 2 material). Indicate which two problems you are omitting: _____ and _____.

1. (a) State the definition of the limit of a function (i.e., what does $\lim\limits_{x \to a} f(x) = L$ mean?).

   (b) Prove that if $\lim\limits_{x \to a} f(x) = L$ and $\lim\limits_{x \to a} g(x) = M$, then $\lim\limits_{x \to a}(f(x) + g(x)) = L + M$ **(1,2)**

2. Do each of the following **(5)**:

   (a) State the Intermediate Value Theorem.

   (b) Use the Intermediate Value Theorem to *prove* that $f(x) = x^5 - 5x - 1$ has a real root (be sure to include all necessary hypotheses).

3. Do each of the following **(12, 18)**:

   (a) State the Extreme Value Theorem.

   (b) Show that the largest area rectangle of a given perimeter is in fact a square.

4. (a) State the Fundamental Theorem of Calculus (both parts). **(23)**

   (b) Evaluate $\displaystyle\int_1^e \ln x \, dx$ (HINT: Use parts) and indicate with a star (*) where you have used the Fundamental Theorem of Calculus in your computations. **(24, 31)**

5. (a) State the definition of *partition*, *norm* of a partition, *Riemann sum*, and *definite integral* for $\displaystyle\int_a^b f(x) \, dx$. **(21)**

   (b) Explain the difference between a definite integral and an indefinite integral (if any). **(20, 23)**

6. Evaluate **(37, 38, 39)**:

(a) $\displaystyle\int_{-\infty}^{\infty} \frac{1}{1+x^2}\, dx$ (use all notation correctly and don't write things that don't make sense).

(b) Evaluate $\displaystyle\int_{-\infty}^{\infty} \frac{1}{x^2}\, dx$.

7. (a) State L'Hôpital's Rule for an ∞/∞ indeterminate form.

(b) Use L'Hôpital's Rule to show $\displaystyle\lim_{x\to\infty} \left(1 + \frac{1}{x}\right)^x = e.$ **(31, 37)**

8. Do each of the following **(41,45)**:

(a) State the definition of the limit of a sequence: $\displaystyle\lim_{n\to\infty} a_n = L$.

(b) State the definition of the sum of a series: $\displaystyle\sum_{n=1}^{\infty} a_n = S$.

(c) Determine whether the series $\displaystyle\sum_{n=1}^{\infty} ne^{-2n}$ is convergent or divergent. You may use any test, but you must check the hypothesis of any test you use.

9. Do each of the following

(a) For a given x value, the power series $\displaystyle\sum_{n=0}^{\infty} c_n(x-a)^n$ may converge conditionally, converge absolutely, or diverge. Describe the possible behavior of this series (i.e. on what types of sets might the series converge conditionally, converge absolutely, or diverge)?

(b) What is the radius of convergence of $\displaystyle\sum_{n=0}^{\infty} \frac{3^n x^n}{n!}$? **(46)**

10. Compute a Taylor series for e^{-x^2} and $\displaystyle\int_0^x e^{-t^2}\, dt.$ **(47)**