

CALCULUS COMPREHENSIVE EXAM

Spring 2014, Prepared by Dr. Robert A. Beeler

May 2, 2014

NAME _____ STUDENT NUMBER _____

Be clear and **give all details**. Use symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. **No calculators!** You may omit one problem from numbers 1 through 5 (which contain Calculus 1 material) and one problem from numbers 6 through 10 (which contain Calculus 2 material). Indicate which two problems you are omitting: _____ and _____. There is a three hour time limit.

1. Do each of the following:

(a) State the definition of the limit of a function (that is, what does $\lim_{x \rightarrow a} f(x) = L$ mean?).

(b) Prove that if $\lim_{x \rightarrow a} f(x) = L$ and $\lim_{x \rightarrow a} g(x) = M$, then $\lim_{x \rightarrow a} (5f(x) + 7g(x)) = 5L + 7M$. (1)

2. Do each of the following (5, 13):

(a) State the Intermediate Value Theorem.

(b) State the Mean Value Theorem.

(c) Prove that $f(x) = 3\sin(x) - 5x + 2$ has a unique real zero.

3. Consider Let $f(x) = \sqrt[3]{x}(x + 7)^2$. Find the first and second derivative of f , find where f is increasing/decreasing, find where f is concave up/concave down, and find the extrema of f . (8, 14, 15, 16, 17)

4. Two cars leave from the same intersection at the same time. The first car is traveling south at a speed of 65 mph. The second car is traveling east at 40 mph. How fast is the distance between the cars when the first car is 13 miles south of the intersection and the second car is 8 miles east of the intersection. (19)

5. Do each of the following(23, 24, 35):

(a) State the two parts of the Fundamental Theorem of Calculus.

(b) Use the Fundamental Theorem of Calculus to evaluate $\int_1^e x \ln(x) dx$ and indicate with a star (*) where you are applying the Fundamental Theorem.

6. Consider the region enclosed by the curves $y = x^2 + x + 5$ and $y = 2x + 17$.
- Find the area of the above region. **(25)**
 - Find the volume of the solid obtained by rotating this region **about the x-axis**. **(26)**
 - SET-UP, BUT DO NOT EVALUATE** an integral that computes the volume of the solid obtained by rotating this region **about the line $x = 5$** . **(26)**
7. (a) State the definition of *partition*, *norm* of a partition, *Riemann sum*, and *definite integral* for $\int_a^b f(x) dx$. **(21)**
- (b) Explain the difference between a definite integral and an indefinite integral (if any). **(20, 23)**
8. Do each of the following **(41, 43)**:
- Let $\{a_n\} = \{a_1, a_2, a_3, \dots\}$ be a sequence of real numbers. Define “ $\lim_{n \rightarrow \infty} a_n = L$.”
 - Use the Integral Test to show that the series $\sum_{n=1}^{\infty} \frac{e^n}{1 + e^{2n}}$ converges.
9. Do each of the following **(46)**:
- For a given x value, the power series $\sum_{n=0}^{\infty} c_n(x - a)^n$ may converge conditionally, converge absolutely, or diverge. Describe the possible behavior of this series (i.e. on what types of sets might the series converge conditionally, converge absolutely, or diverge?).
 - What is the interval of convergence for $\sum_{n=0}^{\infty} \frac{(-1)^n(x + 7)^n}{n + 8}$ (give detailed reasons for your answer).
10. Do each of the following **(44, 47)**:
- Use the MacLaurin series for $\cos(x)$ to find a series for $\int \cos(x^5) dx$.
 - Estimate $\int_0^{\sqrt[5]{\pi}} \cos(x^5) dx$ to the nearest 0.001 and explain why you know your answer has this level of accuracy.