CALCULUS COMPREHENSIVE EXAM

Spring 2019b, Prepared by Dr. Robert Gardner

April 12, 2019

NAME ______________________________ Start Time ________ End Time: ________

Be clear and **give all details**. Use all symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. **No calculators and turn off your cell phones!** Use the paper provided and **only write on one side.**

You may omit one problem from numbers 1 through 5 (which contain Calculus 1 material) and one problem from numbers 6 through 10 (which contain Calculus 2 material). Indicate which two problems you are omitting: ______ and ______.

1. Do each of the following **(1)**:

(a) State the definition of the limit of a function (i.e., what does $\lim_{x\to a} f(x) = L$ mean?).

(b) Use the definition of limit to prove that $\lim_{x\to a}(mx+b) = ma+b$, where $m \neq 0$.

2. Do each of the following **(3)**:

(a) State the Sandwich Theorem (also called the Squeeze Theorem) for the limit of a function.

(b) Use the fact that $\sin(\theta) < \theta < \tan(\theta)$ for $\theta \in (0, \pi/2)$ to show that $\lim_{\theta\to 0}\frac{\sin(\theta)}{\theta} = 1$. WARNING: This is a two sided limit and the inequality is known to hold only for $\theta \in (0, \pi/2)$.

3. Do each of the following **(29,37)**:

(a) State L'Hôpital's rule.

(b) Determine $\lim_{x\to 0^+}(1-2x)^{3/x}$.

4. Do each of the following **(23, 24)**:

(a) State the two parts of the Fundamental Theorem of Calculus.

(b) Evaluate $\int_0^1 \tan^{-1}(x)\,dx$ (HINT: Use parts) and indicate with a star (*) where you have used the Fundamental Theorem of Calculus in your computations. **(24, 31)**

5. (a) State the definition of *partition*, *norm* of a partition, *Riemann sum*, and *definite integral* for $\int_a^b f(x)\,dx$. **(21)**

(b) Explain the difference between a definite integral and an indefinite integral (if any). **(20, 23)**

6. **(a)** Use the definition of $y = \sin^{-1} x$ (in terms of the sine function) and implicit differentiation to find $y' = \dfrac{d}{dx}[\sin^{-1} x]$. **(35)**

(b) Evaluate $\displaystyle\int \frac{dx}{\sqrt{5 - 4x - x^2}}$. **(28, 34, 35)**

7. Do each of the following **(32, 37)**:

(a) If f is continuous on $[a, c) \cup (c, b]$ then state the definition of $\int_a^b f(x)\,dx$. That is, how do you integrate over a discontinuity? You may assume the usual definition for integrals of continuous functions has been established.

(b) Evaluate $\displaystyle\int_0^2 \frac{1}{(x-1)^2}\,dx$.

8. Do each of the following **(41,43, 45)**:

(a) State the definition of the limit of a sequence: $\displaystyle\lim_{n\to\infty} a_n = L$.

(b) State the definition of the sum of a series: $\displaystyle\sum_{n=1}^{\infty} a_n = S$.

(c) Use the integral test to show that the harmonic series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ diverges.

9. Do each of the following **(46)**:

(a) For a given x value, the power series $\displaystyle\sum_{n=0}^{\infty} c_n(x-a)^n$ may converge conditionally, converge absolutely, or diverge. Describe the possible behavior of this series (i.e., on what types of sets might the series converge conditionally, converge absolutely, or diverge)?

(b) What is the radius of convergence of $\displaystyle\sum_{n=0}^{\infty} \frac{3^n x^n}{n!}$?

10. Compute a MacLaurin series for e^{-x^2} and $\displaystyle\int_0^x e^{-t^2}\,dt$. **(47)**