CALCULUS COMPREHENSIVE EXAM

Fall 2016b, Prepared by Dr. Robert Gardner

April 22, 2016

NAME ________________________________ Start Time ________ End Time: ________

Be clear and **give all details**. Use all symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. **No calculators and turn off your cell phones!** Use the paper provided and **only write on one side.** You may omit one problem from numbers 1 through 5 (which contain Calculus 1 material) and one problem from numbers 6 through 10 (which contain Calculus 2 material). Indicate which two problems you are omitting: ______ and ______.

1. **(a)** State the definition of the limit of a function (i.e., what does $\lim_{x\to a} f(x) = L$ mean?).

(b) Prove that if $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$, then $\lim_{x\to a}(f(x) + g(x)) = L + M$ **(1,2)**

2. **(a)** Suppose the interval $[a, b]$ is a subset of the domain of the function f. What is the definition of "f is continuous at point c where $c \in (a, b)$"? What is the definition of "f is continuous on the interval $[a, b]$"? **(4)**

(b) Use the definition above to show that $f(x) = \sqrt{1 - x^2}$ is continuous on $[-1, 1]$. **(4)**

3. Do each of the following **(12, 18)**:

(a) State the Extreme Value Theorem.

(b) Show that the largest area rectangle of a given perimeter is in fact a square.

4. **(a)** State the Fundamental Theorem of Calculus (both parts). **(23)**

(b) Evaluate $\int_1^e \ln x\, dx$ (HINT: Use parts) and indicate with a star (*) where you have used the Fundamental Theorem of Calculus in your computations. **(24, 31)**

5. **(a)** State the definition of *partition*, *norm* of a partition, *Riemann sum*, and *definite integral* for $\int_a^b f(x)\, dx$. **(21)**

(b) Explain the difference between a definite integral and an indefinite integral (if any). **(20, 23)**

6. Evaluate **(37, 38, 39)**:

(a) $\displaystyle\int_{-\infty}^{\infty} \frac{2x}{(1+x^2)^2}\,dx$ (use all notation correctly and don't write things that don't make sense).

(b) Evaluate $\displaystyle\int_{-\infty}^{\infty} \frac{1}{x^2}\,dx$.

7. Do each of the following.

(a) State the definition of $\ln x$ (using integrals). **(29)**

(b) Use the definition from part **(a)** to *prove* that $\ln(ab) = \ln(a) + \ln(b)$. **(29)**

8. Do each of the following **(41,45)**:

(a) State the definition of the limit of a sequence: $\displaystyle\lim_{n\to\infty} a_n = L$.

(b) State the definition of the sum of a series: $\displaystyle\sum_{n=1}^{\infty} a_n = S$.

(c) Determine whether the series $\displaystyle\sum_{n=1}^{\infty} ne^{-2n}$ is convergent or divergent. You may use any test, but you must check the hypothesis of any test you use.

9. Do each of the following

(a) For a given x value, the power series $\displaystyle\sum_{n=0}^{\infty} c_n(x-a)^n$ may converge conditionally, converge absolutely, or diverge. Describe the possible behavior of this series (i.e. on what types of sets might the series converge conditionally, converge absolutely, or diverge)?

(b) What is the radius of convergence of $\displaystyle\sum_{n=0}^{\infty} \frac{3^n x^n}{n!}$? (**46**)

10. Compute a MacLaurin series for $\sin x^2$ and $\displaystyle\int_0^x \sin t^2\,dt$. (**47**)