CALCULUS COMPREHENSIVE EXAM

Spring 2007b, Prepared by Dr. Yared Nigussie

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NAME________________________ STUDENT NUMBER____________________________________

Be clear and **give all details**. Use symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide.

You may omit one problem from numbers 1 through 5 (which contain Calculus 1 material) and one problem from numbers 6 though 10 (which contain Calculus 2 material). Indicate which two problems you are omitting: ______ and ______. There is a three hour time limit.

1. (a) State the definition of the limit of a function (i.e., what does $\lim_{x\to a} f(x) = L$ mean?).

(b) Prove that if $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$, then $\lim_{x\to a}(f(x) + g(x)) = L + M$ **(1,2)**

2. Prove that if $f(x)$ has a derivative at $x = c$, then f is continuous at $x = c$. Is the converse also true? **(4, 7)**

3. Do each of the following **(12, 18)**:

(a) State the Extreme Value Theorem.

(b) Show that the largest area rectangle of a given perimeter is in fact a square.

4. (a) State the Fundamental Theorem of Calculus (both parts). **(23)**

(b) Evaluate $\int_1^e \ln x\, dx$ (HINT: Use parts) and indicate with a star (*) where you have used the Fundamental Theorem of Calculus in your computations. **(24, 31)**

5. (a) State the definition of *partition*, *norm* of a partition, *Riemann sum*, and *definite integral* for $\int_a^b f(x)\, dx$. **(21)**

(b) Explain the difference between a definite integral and an indefinite integral (if any). **(20, 23)**

6. Do each of the following **(37, 38, 39)**:

(a) State the definition of $\ln(x)$ (using integrals).

(b) Use the definition from part **(a)** to prove that $\ln(ab) = \ln(a) + \ln(b)$.

7. The region in the first quadrant bounded by $y = x^2$, the y-axis, and the line $y = 1$ is revolved about the line $x = 2$. Find the resulting volume. **(26)**

8. State the Integral Test (which concerns the convergence of a positive term series). Show that for $0 < p < 1$ the p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ diverges. **(38,43)**

9. Do each of the following **(41)**

(a) State the definition of the limit sequence: $\lim_{n \to \infty} a_n = L$.

(b) State the definition of the sum of a series $\sum_{n=1}^{\infty} a_n = S$.

(c) Evaluate $\sum_{n=1}^{\infty} (1 - \frac{1}{2^n})$.

10. Find a Maclaurin Series for $f(x) = e^x$ (show your work). Where does the series converge absolutely? Where does it converge conditionally? Where does it diverge? Use the series to verify that $\int e^x dx = e^x + C$. **(31, 45, 46, 47)**