CALCULUS COMPREHENSIVE EXAM

Spring 2020a, Prepared by Dr. Robert Gardner

February 7, 2020

NAME ______________________________ Start Time ________ End Time: ________

Be clear and **give all details**. Use all symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. **No calculators and turn off your cell phones!** Use the paper provided and **only write on one side.**

You may omit one problem from numbers 1 through 5 (which contain Calculus 1 material) and one problem from numbers 6 through 10 (which contain Calculus 2 material). Indicate which two problems you are omitting: ______ and ______.

1. (a) State the definition of the limit of a function (i.e., what does $\lim_{x\to a} f(x) = L$ mean?).

(b) Prove that if $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$, then $\lim_{x\to a}(f(x) - g(x)) = L - M$ **(1,2)**

2. Prove that if $f(x)$ has a derivative at $x = c$, then f is continuous at $x = c$. Is the converse also true? **(4, 7)**

3. Do each of the following **(8, 10, 31, 35)**:

(a) State the Chain Rule (with all hypotheses).

(b) What does it mean for $f(x)$ to be implicit to the equation $F(x, y) = 0$.

(c) Find $\dfrac{dy}{dx} : \tan^{-1}(\ln y) = e^{x^2}$.

4. (a) State the Fundamental Theorem of Calculus (both parts). **(23)**

(b) Evaluate $\int_1^2 xe^x \, dx$ and indicate with a star (*) where you have used the Fundamental Theorem of Calculus in your computations. **(24, 31)**

5. (a) State the definition of *partition*, *norm* of a partition, *Riemann sum*, and *definite integral* for $\int_a^b f(x)\, dx$. **(21)**

(b) Explain the difference between a definite integral and an indefinite integral (if any). **(20, 23)**

6. Find the length of $y = x^2$ for $x \in [0, 1]$ **(23)**.

7. Do each of the following. Respect the calculus!

(a) Evaluate $\lim_{x\to 0^+} \left(1 + \dfrac{1}{x}\right)^x$. **(31, 37)**

(b) Evaluate $\int_{-1}^1 \dfrac{1}{x^2}\, dx$. **(39)**

8. Do each of the following.

(a) State the definition of the limit of a sequence: $\lim_{n\to\infty} a_n = L$. **(41)**

(b) State the definition of the sum of a series: $\sum_{n=1}^{\infty} a_n = S$. **(41)**

(c) Evaluate $\sum_{n=1}^{\infty} \left(1 - \frac{1}{2^n}\right)$. **(45)**

9. Do each of the following:

(a) For a given x value, the power series $\sum_{n=0}^{\infty} c_n(x-a)^n$ may converge conditionally, converge absolutely, or diverge. Describe the possible behavior of this series (i.e. on what types of sets might the series converge conditionally, converge absolutely, or diverge)? **(46)**

(b) What is the radius of convergence of $\sum_{n=0}^{\infty} \frac{3^n x^n}{n!}$?**(46)**

10. Compute a MacLaurin series for $\sin(-x^3)$ and $\int_0^x \sin(-t^3)\,dt$. (**47**)