CALCULUS COMPREHENSIVE EXAM

Spring 2007, Prepared by Dr. Robert Gardner

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NAME ______________________ STUDENT NUMBER __________________________________

Be clear and **give all details**. Use symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. **No calculators!** You may omit one problem from numbers 1 through 5 (which contain Calculus 1 material) and one problem from numbers 6 though 10 (which contain Calculus 2 material). Indicate which two problems you are omitting: ______ and ______. There is a three hour time limit.

1. Do each of the following **(1)**:

(a) State the definition of the limit of a function (i.e., what does $\lim_{x\to a} f(x) = L$ mean?).

(b) Use the definition of limit to prove that $\lim_{x\to -2} -3x + 1 = 7$.

2. Do each of the following **(8, 10, 31, 35)**:

(a) State the Chain Rule (with all hypotheses).

(b) What does it mean for $f(x)$ to be implicit to the equation $F(x, y) = 0$.

(c) Find $\frac{dy}{dx}$: $\tan^{-1}(\ln y) = e^{x^2}$.

3. Do each of the following **(12, 18)**:

(a) State the Extreme Value Theorem.

(b) Show that the largest area rectangle of a given perimeter is in fact a square.

4. (a) State the Fundamental Theorem of Calculus (both parts). **(23)**

(b) Evaluate $\int_1^e \ln x\, dx$ (HINT: Use parts) and indicate with a star (*) where you have used the Fundamental Theorem of Calculus in your computations. **(24, 31)**

5. (a) State the definition of *partition*, *norm* of a partition, *Riemann sum*, and *definite integral* for $\int_a^b f(x)\, dx$. **(21)**

(b) Explain the difference between a definite integral and an indefinite integral (if any). **(20, 23)**

6. Find the length of $y = x^2$ for $x \in [0,1]$. **(24, 27, 34)**

7. Do each of the following **(37, 38, 39)**:

(a) Evaluate $\lim\limits_{x \to 0^+} x^x$.

(b) Evaluate $\displaystyle\int_{-\infty}^{\infty} \frac{1}{x^2}\,dx$.

8. (a) Let $\{a_n\} = \{a_1, a_2, a_3, \ldots\}$ be a sequence of real numbers. Define "$\lim\limits_{n\to\infty}(a_n) = L$." **(41)**

(b) Let $\displaystyle\sum_{n=1}^{\infty} a_n$ be a series. Define *partial sum* of the series and define "$\displaystyle\left(\sum_{n=1}^{\infty} a - n\right) = L$." **(41)**

9. Determine whether the following series converge or diverge and explain. **(43)**

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1}$.

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(2n+1)!}$.

10. Do each of the following **(46)**:

(a) For a given x value, the power series $\displaystyle\sum_{n=0}^{\infty} c_n(x-a)^n$ may converge conditionally, converge absolutely, or diverge. Describe the possible behavior of this series (i.e., on what types of sets might the series converge conditionally, converge absolutely, or diverge)?

(b) What is the radius of convergence of $\displaystyle\sum_{n=0}^{\infty} \frac{3^n x^n}{n!}$?