CALCULUS COMPREHENSIVE EXAM

Spring 2016a, Prepared by Dr. Robert Gardner

January 29, 2016

NAME ______________________________ Start Time ________ End Time: ________

Be clear and **give all details**. Use all symbols correctly (such as equal signs) and write in complete sentences. The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. **No calculators and turn off your cell phones!** Use the paper provided and **only write on one side.** You may omit one problem from numbers 1 through 5 (which contain Calculus 1 material) and one problem from numbers 6 through 10 (which contain Calculus 2 material). Indicate which two problems you are omitting: ______ and ______.

1. **(a)** State the definition of the limit of a function (i.e., what does $\lim_{x\to a} f(x) = L$ mean?).

(b) Prove that if $\lim_{x\to a} f(x) = L$ then, for all $k \in \mathbb{R}$, $\lim_{x\to a} kf(x) = kL$. **(1,2)**

2. **(a)** Suppose the interval $[a, b]$ is a subset of the domain of the function f. What is the definition of "f is continuous at point c where $c \in (a, b)$"? What is the definition of "f is continuous on the interval $[a, b]$"? **(4)**

(b) Use the definition above to show that $f(x) = \sqrt{1 - x^2}$ is continuous on $[-1, 1]$. **(4)**

3. **(a)** What does it mean for $f(x)$ to be implicit to the equation $F(x, y) = 0$? **(10)**

(b) Find y' if $\sin(xy) = \ln(x \cot y)$. **(8, 31, 34)**

4. **(a)** State the Fundamental Theorem of Calculus (both parts). **(23)**

(b) Evaluate $\int_{-1}^{1} \frac{1}{1+x^2}\, dx$ and indicate with a star $(*)$ where you have used the Fundamental Theorem of Calculus in your computations. **(24, 35)**

5. **(a)** State the definition of *partition*, *norm* of a partition, *Riemann sum*, and *definite integral* for $\int_a^b f(x)\, dx$. **(21)**

(b) Evaluate $\int_{-1}^{1} \frac{1}{x^2}\, dx$. Use correct notation. **(38, 39)**

6. Do each of the following.

(a) State the definition of $\ln x$ (using integrals). **(29)**

(b) Use the definition from part **(a)** to *prove* that $\ln(ab) = \ln(a) + \ln(b)$. **(29)**

7. Do each of the following.

(a) State the definition of the limit of a sequence: $\lim_{n\to\infty} a_n = L$. **(41)**

(b) State the definition of the sum of a series: $\sum_{n=1}^{\infty} a_n = S$. **(41)**

(c) Evaluate $\sum_{n=1}^{\infty}\left(1 - \frac{1}{2^n}\right)$. **(42)**

8. **(a)** State the Integral Test (which concerns the convergence of a positive term series).

(b) Prove that for $p > 1$ the p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges. **(38, 43)**

9. Do each of the following:

(a) For a given x value, the power series $\sum_{n=0}^{\infty} c_n(x-a)^n$ may converge conditionally, converge absolutely, or diverge. Describe the possible behavior of this series (i.e. on what types of sets might the series converge conditionally, converge absolutely, or diverge)? **(46)**

(b) For what values of x does $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n}$ converge? **(46)**

10. Compute a MacLaurin series for $\sin x^2$ and $\int_0^x \sin t^2\, dt$. (**47**)