CALCULUS COMPREHENSIVE EXAM
SPRING 1999, Prepared by Dr. Robert Gardner

NAME ______________________________ STUDENT NUMBER ________________________
Be clear and **give all details**. Use all symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide.

You may omit one problem from numbers 1 through 6 (which contain material from Calculus 1) and one problem from numbers 7 through 12 (which contain material from Calculus 2). Indicate which two problems you are omitting: ______ and ______.

1. Do each of the following **(4)**:

a. What does it mean for function f to be continuous at point a?

b. What does it mean for f to be continuous on interval $[a, b]$?

c. Show that $f(x) = \sqrt{1 - x^2}$ is (or is not) continuous on $[-1, 1]$.

2. Let f be a function defined on an open internal containing point c. Prove that if f is differentiable at c, then f is continuous at c. **(7)**

3. Do each of the following **(10)**:

a. What does it mean for $y = f(x)$ to be a function *implicit* to the equation $F(x, y) = 0$?

b. Find the equation of the line tangent to $x^2 - xy + y^2 = 7$ at the point $(-1, 2)$.

4. Do each of the following **(15, 16, 17)**:

a. Consider $f(x) = \dfrac{x^2}{x^2 - 1}$.

b. On what intervals is f concave up and on what intervals is f concave down?

c. What are the asymptotes of f?

d. Graph f?

5. A man 6 feet tall walks at a rate of 5 feet/second toward a streetlight that is 16 feet above the ground. At what rate is the tip of his shadow moving? At what rate is the length of his shadow changing when he is 10 feet from the base of the light? **(19)**

6. Do each of the following **(23, 24)**:

a. State the two parts of the Fundamental Theorem of Calculus.

b. Use the Fundamental Theorem of Calculus to evaluate $\int_0^1 x \sin x \, dx$ and indicate with a star $(*)$ where you are applying the Fundamental Theorem.

7. Find the length of the curve given by the equation $y = \int_0^x \sqrt{\sec^4 t - 1}\, dt$ for $-\pi/4 \le x \le \pi/4$. **(23, 27)**

8. Do each of the following **(29)**:

a. State the definition of $\ln x$ (using integrals).

b. Use the definition to *prove* that: $\ln xy = \ln x + \ln y$.

9. Do each of the following **(33, 34, 35)**:

a. Evaluate $\lim_{y \to 0} \dfrac{\sin 3y}{4y}$.

b. Evaluate $\lim_{x \to 0^+} \left(1 + \dfrac{1}{x}\right)^x$.

c. Evaluate $\int \dfrac{\sec^2 x\, dx}{\sqrt{1 - \tan^2 x}}$.

10. Do each of the following **(38, 39, 41)**:

a. Evaluate $\int_{-\infty}^{\infty} \dfrac{2x\, dx}{(x^2+1)^2}$.

b. Evaluate $\int_{-1}^{1} \dfrac{1}{x^2}\, dx$.

c. State the definition of *sequence* and what it means for a sequence to *have limit L*.

11. Do each of the following **(45, 46)**:

a. Does the series $\sum_{n=1}^{\infty} n! e^{-n}$ converge or diverge? Give reasons for your answer.

b. For a given X value, the power series $\sum_{n=0}^{\infty} c_n (x-a)^n$ may converge conditionally, converge absolutely, or diverge. Describe the possible behavior of this series (i.e. on what types of sets might the series converge conditionally, converge absolutely, or diverge)?

c. For what values of x does $\sum_{n=1}^{\infty} (-1)^{n-1} \dfrac{x^n}{n}$ converge?

12. Do each of the following **(44, 47)**:

a. Use the MacLaurin series for e^x to find a series for $\int e^{-x^2}\, dx$.

b. Estimate $\int_0^1 e^{-x^2}\, dx$ to the nearest 0.001 and explain why you know your answer has this level of accuracy.