CALCULUS COMPREHENSIVE EXAM
SPRING 1998, Prepared by Dr. Robert Gardner

NAME ______________________________ STUDENT NUMBER ______________________

Be clear and give all details. Use all symbols correctly (such as equal signs). The numbers in bold faced parentheses indicate the number of the topics covered in that problem from the Study Guide. You may omit one problem from numbers 1 through 6 (which contain material from Calculus 1) and one problem from numbers 7 through 12 (which contain material from Calculus 2). Indicate which two problems you are omitting: ______ and ______.

1. Do each of the following **(1)**:

- **a.** Give the definition of "the limit as x approaches a of $f(x)$ is L: $\lim_{x \to a} f(x) = L$."
- **b.** Use the definition to *prove* that $\lim_{x \to 4}(-3x + 5) = -7$.

2. Do each of the following **(4)**:

- **a, b.** Suppose the interval $[a, b]$ is a subset of the domain of function f. What is the definition of "f is continuous at point c where $c \in (a, b)$"? What is the definition of "f is continuous on the interval $[a, b]$"?
- **c.** Use the definitions from **a** and **b** to show that $f(x) = \sqrt{1 - x^2}$ is continuous on $[-1, 1]$.

3. Do each of the following **(8, 10, 31, 34)**:

- **a.** State the Chain Rule (with all hypotheses).
- **b.** What does it mean for $f(x)$ to be implicit to the equation $F(x, y) = 0$?
- **c.** Differentiate (you need not simplify your answer): $f(x) = \ln \sqrt{\dfrac{\cot(e^x)}{\arctan(x) + x^2}}$.

4. Do each of the following **(17)**:

- **a.** Give the definition of "f has a vertical asymptote at $x = a$."
- **b.** Give the definition of "f has a horizontal asymptote of $y = k$."
- **c.** Find the asymptotes of $y = \dfrac{x + 5}{8 - 2x}$ and graph.

5. A 5 foot ladder leans against a wall and the bottom of the ladder is pulled away from the wall at a rate of 1 ft/sec. How fast is the top of the ladder falling when the top of the ladder is 3 feet from the floor? **(19)**

6. Do each of the following **(23)**:

- **a.** State the two parts of the Fundamental Theorem of Calculus.
- **b.** Use the Fundamental Theorem of Calculus to evaluate $\int_0^1 x \sin x \, dx$ and indicate with a star $(*)$ where you are applying the Fundamental Theorem.

7. Do each of the following **(23)**:

a. State the definition of $\ln x$ (using integrals).

b. Use the definition to *prove* that $\ln(ab) = \ln(a) + \ln(b)$.

8. Do each of the following **(24)**:

a. Evaluate $\displaystyle\int \frac{1}{1-x^2}\,dx$.

b. Evaluate $\displaystyle\int \frac{1}{x^2+4x+5}\,dx$.

c. Evaluate $\displaystyle\int \sin^3(x)\cos^3(x)\,dx$.

9. Do each of the following **(32, 37, 39)**:

a. If f is continuous on $[a,c) \cup (c,b]$ then state the definition of $\displaystyle\int_a^b f(x)\,dx$. That is, how do you integrate over a discontinuity? You may assume the usual definition for integrals of continuous functions has been established.

b. Evaluate $\displaystyle\int_0^2 \frac{1}{(x-1)^2}\,dx$.

c. Evaluate $\displaystyle\lim_{x\to 0}\left(\frac{3^x-1}{2^x-1}\right)$.

10. Do each of the following **(41)**:

a. Let $\{a_n\} = \{a_1, a_2, a_3, \ldots\}$ be a sequence of real numbers. Define "$\displaystyle\lim_{n\to\infty}(a_n) = L$."

b, c. Let $\displaystyle\lim_{n=1}^{\infty} a_n$ be a series. Define *partial sum* of the series and define "$\displaystyle\left(\sum_{n=1}^{\infty} a_n\right) = L$."

11. Do each of the following **(43, 44, 48)**

a. Does $\displaystyle\sum_{n=2}^{\infty} \frac{\ln n}{n}$ converge?

b. Approximate $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ to the nearest 0.1. How do you know your approximation is accurate to the nearest 0.1?

c. Find the radius of convergence for $\displaystyle\sum_{n=0}^{\infty} \frac{3^n x^n}{n!}$.

12. Compute a Taylor series for e^{-x^2} and $\displaystyle\int_0^x e^{-t^2}\,dt$.