STUDY GUIDE FOR CALCULUS COMPREHENSIVE EXAM

Department of Mathematics, M.S. Program
Prepared by Dr. Robert Gardner
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Statement of Purpose: The purpose of the comprehensive calculus exam is to force students to recall material from freshman calculus. By reviewing this material, it is expected that the math graduate school experience will be enhanced. Many graduate level classes will require a working knowledge of such topics as differentiation, integration, and modeling, and this test is intended to refresh your knowledge of these topics.

When tests are given: The calculus comprehensive exam will be offered twice each Fall and Spring semester. The test will be given either during the week before classes start or in the first two weeks of classes, and sometime shortly after the middle of each term. Assuming demand, the test will also be offered once over the summer. You will choose 8 questions from a list of 10 and have 3 hours to complete the test.

Material possible covered: Questions on the test will be drawn from the following topics. Expect both theoretical and computational questions.

1. Definition of limit and application to functions of type $f(x) = mx + b$.

2. Properties of limits (sums, differences, products. . .).

3. State and use of the Sandwich Theorem.

4. Definition of continuity at a point, on a closed and bounded interval, and use.

5. Intermediate Value Theorem and application to proving existence of roots.

6. Definition of derivative and its use to calculate derivatives.

7. Proof of "Differentiable implies continuous."

8. Calculation of derivatives and statement of differentiation rules.

9. Use of differentials to approximate errors.

10. Implicit differentiation and "What does it mean for a function to be implicit to an equation?"

11. Definition of increasing and decreasing and of maximum and minimum.

12. Finding absolute extrema on closed, bounded intervals. Extreme Value Theorem.

13. Rolle's Theorem and the Mean Value Theorem (for derivatives).

14. Relationship between increasing/decreasing and the first derivative; applications to graphing.

15. Relationship between concave up/down and the second derivative; application to graphing.

16. Definition of critical point, point of inflection, and method to find them.

17. Definition of horizontal and vertical asymptotes and how to find them for a given function.

18. MAX/MIN problems.

19. Related rates problems.

20. Antiderivatives and indefinite integrals.

21. Definition of *partition* of $[a, b]$, *norm* (or "mesh") of a partition, *Riemann sum* and *definite integral*. Application of the definition to use a regular partition to compute a definite integral for a given function.

22. Mean Value Theorem for integrals and its use.

23. Statement of Fundamental Theorem of Calculus (two parts) and its use to evaluate definite integrals.

24. Methods of integration: u-substitution, integration by parts, trig substitution, partial fractions, trig integrals (i.e. integrals of powers of trig functions).

25. Area between curves.

26. Solids of revolution: washers, shells, and slicing.

27. Applications of integrals: work, force of a liquid, arc length.

28. Definition of inverse functions and finding inverse for a given function.

29. Definition of natural log as a definite integral, properties of $\ln x$ and their proofs using the definition of $\ln x$.

30. e^x and properties.

31. Differentiation of $\ln x$ and e^x.

32. Definition of a^x for $x > 0$.

33. Limits of sin and cos, such as $\lim_{x\to 0} \dfrac{\sin x}{x}$ and $\lim_{x\to 0} \dfrac{1-\cos x}{x}$.

34. Derivatives and integrals of trig functions.

35. Definition of inverse trig functions, their derivatives, and integrals which yield inverse trig functions.

36. Centroids of regions, moments.

37. Indeterminant forms, L'Hopital's Rule and applications.

38. Improper integrals and definition of $\int_0^\infty f(x)\,dx$.

39. Integrals over discontinuities.

40. Statement of Taylor's formula.

41. Definition of sequence, limit of a sequence, definition of series, partial sums and sum of a series. Definition of convergent and divergent series.

42. Properties of series (sum, difference,...), "Test for Divergence."

43. Positive term series: integral test, harmonic series, p-series, comparison text, limit comparison test.

44. Alternating series: definition, test for convergence, approximating the sum.

45. Absolute convergence, conditional convergence, ratio test, root test.

46. Definition of power series, convergence properties of power series and radius of convergence. Calculation of radius of convergence.

47. Computation of Taylor and MacLaurin series.

48. Computation of double integrals over rectangles and Fubini's Theorem (First Form).

49. Computation of double integrals over general regions and Fubini's Theorem (Stronger Form), using horizontal/vertical cross-sections, finding areas using double integrals, average value of $f(x, y)$ over a region.

50. Double integrals in polar form, finding areas using double integrals in polar form, changing double integrals from Cartesian form to polar form, showing the total area under the standard normal distribution is 1.