Significance tests for a Proportion - Large-Sample Test

ASSUMPTIONS FOR INFERENCE ABOUT A PROPORTION

- The data are an SRS from the population of interest

- The population is at least 10 times as large as the sample

- For a test of $H_0 : p = p_0$, the sample size n is so large that both np_0 and $n(1 - p_0)$ are 10 or more.

To test the hypothesis $H_0 : p = p_0$, compute the z statistic

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

In terms of a variable Z having the standard Normal distribution, the approximate P-value for a test of H_0 against

$$
\begin{array}{lll}
H_a : p > p_0 & \text{is} & P(Z \geq z) \\
H_a : p < p_0 & \text{is} & P(Z \leq z) \\
H_a : p \neq p_0 & \text{is} & 2P(Z \geq |z|)
\end{array}
$$

Example 1: In 1993 the General Social Survey found that approximately 23% of the adult population opposed the death penalty for persons convicted of murder. A researcher thinks that the current proportion of the adult population that opposes the death penalty is greater than 23%. The researcher takes an SRS of 2000 adults and finds that 535 people oppose the death penalty. State the hypotheses, give the test statistic, find the P-value, and state your conclusion. Use the significance level $\alpha = 0.01$.

The sample proportion that oppose the death penalty is $\hat{p} = \frac{535}{2000} = 0.2675$.

Step 1. Hypotheses.
$$H_0 : p = .23 \text{ vs. } H_a : p > .23$$

Step 2. Test Statistic.

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

$$= \frac{.2675 - .23}{\sqrt{\frac{(.23)(.67)}{2000}}}$$

$$= 3.99$$

Step 3. P-value. The P-value is the area on the standard Normal curve more than 3.99. This value is off the chart. Hence, the area is approximately 0.

Step 4. Conclusion. Since the P-value is approximately 0 and it is smaller than the given significance level $\alpha = 0.01$ we reject the null hypothesis ($H_0 : p = .23$). Thus, there is strong evidence (P-value is really small) that the proportion of the adult population that oppose the death penalty is greater than 23%.

Note that the assumptions for the above large-sample test are satisfied.

Example 2: (Source: Intro Stats by De Veaux and Velleman) An airline's public relations department says that the airline rarely loses Passenger's luggage. It further claims that on those occasions when luggage is lost, 90% is recovered and delivered to its owner within 24 hours. A consumer group who surveyed a large number of air travelers found that only 103 of 122 people who lost luggage on that airline were reunited with the missing items by the next day. Does this cast doubt on the airline's claim? Explain.

The sample proportion who lost their luggage is $\hat{p} = \frac{103}{122} = 0.8443$.

Step 1. Hypotheses.
$$H_0 : p = .90 \text{ vs. } H_a : p < .90$$

Step 2. Test Statistic.
$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

$$= \frac{.8443 - .90}{\sqrt{\frac{(.90)(.10)}{122}}}$$

$$= -2.05$$

Step 3. P-value. The P-value is the area on the standard Normal curve less than -2.05. Hence, the area is approximately 0.020.

Step 4. Conclusion. The approximate P-value = 0.020 says that if the true proportion of recovered lost luggage were .90, then the observed value of 0.8443 (or smaller) would occur about 2 out of 100 (2%) times. This suggests that the airline recovers lost luggage within 24 hours less than the claimed 90%.

Note that the assumptions for the above large-sample test seem to be okay. We have $np_0 = (122)(.90) = 109.8 > 10$ and $n(1 - p_0) = (122)(.1) = 12.2 > 10$ (this is close). We are assuming that the consumer group conducted an SRS. It seems reasonable that the population size (the number of air travelers) is at least 10 times the 122 people sampled.

Example 3: (Source: The Practice of Business Statistics by Moore et al.) A national survey of restaurant employees found that 75% said that work stress had a negative impact on their personal lives. A sample of 100 employees of a restaurant chain finds that 68 answer 'Yes" when asked, "Does work stress have a negative impact on your personal life?" Is this good reason to think that the proportion of all employees of this chain who would say "Yes" differs from the national proportion? Explain.

The sample proportion who said "Yes" when asked, "Does work stress have a negative impact on your personal life?" is $\hat{p} = \frac{68}{100} = 0.68$.

Step 1. Hypotheses.
$$H_0 : p = .75 \text{ vs. } H_a : p \neq .75$$

Step 2. Test Statistic.
$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

$$= \frac{.68 - .75}{\sqrt{\frac{(.75)(.25)}{100}}}$$

$$= -1.62$$

Step 3. P-value. We find $P(Z \leq -1.62) = 0.0526$. The P-value is the area in both tails,
$$P - value = 2P(Z \geq |-1.62|) = 2 \times 0.0526 = 0.1052..$$

Step 4. Conclusion. We fail to reject. We conclude that the restaurant chain employees are compatible with the national survey results.

Note that the assumptions for the above large-sample test seem to be okay. We have $np_0 = (100)(.75) = 75 > 10$ and $n(1 - p_0) = (100)(.25) = 25 > 10$. We are assuming that the survey conducted an SRS. We are also assuming that the number of employees that work for the restaurant chain is at least 1000.